Current distribution in Hall bars and breakdown of the quantum Hall effect

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Abstract

A numerical study is made of current distribution in small Hall bars with disorder. It is observed, in particular, that in the Hall-plateau regime the Hall current tends to concentrate near the sample edges while it diminishes on average in the sample interior as a consequence of localization. Also reported is another numerical experiment on a related, but rather independent topic, the breakdown of the quantum Hall effect. It is pointed out that the competition of the Hall field with disorder in the sample interior, an *intra*subband process, can account for both the magnitude and magnetic-field dependence $\propto B^{3/2}$ of the critical breakdown fields observed experimentally.

Keywords: Quantum Hall effect; Current distribution; Breakdown of quantum Hall effect

1. Introduction

The current distribution in a Hall sample is a subject of interest pertaining to the foundation of the quantum Hall effect (QHE). In spite of various studies, both experimental [1] and theoretical [2-6], it is still a focus of attention and controversy whether the Hall current flows in the sample interior or along the sample edges.

In this paper we first report on a numerical study of current distribution in small Hall samples with disorder and verify some theoretical observations [7]. In particular, we demonstrate an important physics of redistribution of the Hall current via disorder.

A by-product of the numerical analysis is a hint as to a rather independent subject, the breakdown of the quantum Hall effect [8-13]. We report on a numerical analysis [14] to verify that the competition of the Hall field with disorder in the sample interior, an *intra*subband process, can account for both the magnitude and magnetic-field dependence $\propto B^{3/2}$ of the critical breakdown fields E^{cr} observed in a series of experiments by Kawaji *et al.* [11-13].

2. Method of calculation

Consider a Hall bar bent into a loop of circumference L_x and width L_y, with a uniform magnetic field B normal to the plane. We distribute short-range impurities over it through an impurity potential $U(x,y) = \sum_i \lambda_i \, \delta(x - x_i)\delta(y - y_i)$, and detect the current j_x flowing in response to a uniform Hall field E_y. We take the $y = 0$ edge to be a sharp edge where the wave function is bound to vanish and leave the other edge $y = L_y$ as a gentle edge.

For numerical calculations it is advantageous to work with the basis $\{|n, y_0\rangle\}$ of the eigenstates of the $U = E_y = 0$ case, labeled by $n = 0, 1, 2, \cdots$, and $y_0 = \ell^2 p_x$, with $\ell \equiv 1/\sqrt{eB}$. Diagonalizing the Hamiltonian with respect to Landau labels n by a unitary transformation W, one obtains the Hamiltonian governing each impurity-broadened Landau subband:

$$
\begin{aligned}
H_n^W(y_0, y_0') =\,& h(y_0)\,\delta_{y_0 y_0'} + U_{nn}(y_0, y_0') + \cdots, \\
h(y_0) =\,& \omega\{\nu_n(y_0) + \tfrac{1}{2}\} + eE_y\{y_0 - \ell^2 \nu_n{}'(y_0)\},
\end{aligned}
\tag{1}
$$

where the confining potentials $\omega \nu_n(y_0)$ relevant around $y_0 \sim 0$ derive from the sharp edge.

Numerically diagonalizing H_n^W, one obtains eigenstates $|\alpha\rangle$ forming the nth subband and can calculate the associated current density $j_x^{(\alpha)}(x, y)$. We shall distinguish between $j_x^{(\alpha)}(x, y)$ present even in equilibrium and its response to a Hall field, the Hall-current component $j_{\text{Hall}}^{(\alpha)}(x, y) = j_x^{(\alpha)}(x, y | E_y + \delta E_y) - j_x^{(\alpha)}(x, y | E_y)$, calculated with $\delta E_y = E_y/100$.

There are three competing effects to be considered. (i) Impurities capture electrons and make them localized. In contrast, (ii) a Hall field makes them drift with velocity $v_x \sim E_y/B$. A simple estimate of energy cost reveals how these two effects compete: An electron state localized around an isolated impurity of strength λ acquires an energy shift $\triangle\epsilon \approx \lambda/(2\pi\ell^2) \equiv s\,\omega$, where we have introduced a dimensionless strength s. When a Hall field is turned on, it will perceive over its spatial extent of $O(\ell)$ an energy variation of magnitude $\sim e\ell\,E_y$. Accordingly, if the field becomes so strong that the field-to-disorder ratio

$$R \equiv e\ell\,|E_y|/(|s|\,\omega) \gtrsim 1, \tag{2}$$

the electron state would be delocalized. The numerical experiment given below reveals that $R^{\text{cr}} \sim O(0.1)$ is the critical value for delocalization. Finally, (iii) the sharp edge $y = 0$ drives electrons along it by an "effective field" as strong as $e\ell E_y^{\text{eff}} \sim \omega\ell\,\nu_n' \sim \omega$.

3. Hall-current distribution

We have examined current distributions for a number of samples. Here we mainly quote the case of a sample of length $\sqrt{2\pi}\ell \times 28 \approx 70\ell$ and width $\sqrt{2\pi}\ell \times 8 \approx 20\ell$, supporting $28 \times 8 = 224$ electron states in the $n = 0$ subband. 180 impurities of varying strength $|s_i| \leq s_{\max} \sim 0.1$ are randomly distributed on it. We choose $E_y < 0$ so that the $y = 0$ edge is the "upper" edge, and in most cases take it very weak, $R = e\ell|E_y|/(s_{\max}\,\omega) \sim 1/10^4$.

We first summarize the observations drawn from our numerical analysis [7]:
(i) The equilibrium current j_x and the Hall current j_{Hall} are substantially different in distribution. (ii) In the Hall-plateau regime the edge states (of the uppermost subband) are vacant and scarcely contribute to the current distribution. This indicates that the edge states are in no sense the principal carriers of the Hall current. (iii) In the plateau regime the Hall current j_{Hall} tends to diminish on average in the sample bulk and concentrate near the sample edges. (iv) The sharp edge and disorder combine to efficiently delocalize electrons near the edge. (v) The Hall field competes with disorder to delocalize electrons in the sample bulk.

Evidence for (i) and (ii) is seen from Fig. 1(a), where the total currents J_x and J_{Hall} carried by the $n = 0$ subband are plotted as a function of vacancies N_v. There the edge states, carrying a large amount of current per state, are readily identified through a rapid decrease of J_x for $0 \leq N_v \lesssim 15$. They, however, have little effect on J_{Hall}.

Note the contrast between the upper and lower plateaus in Fig. 1(a). This suggests observation (iv).

See next the (x-averaged) Hall-current distribution across the sample width in Fig. 1(b), which clearly demonstrates (iii). The density distribution in Fig. 1(c) also shows that in the plateau regime more electrons survive near the sample edge than in the interior. This gives evidence for the expulsion of the Hall current out of the disordered sample bulk, expected theoretically [7]. The decrease of the current in the sample bulk is correlated with the dominance of localized states there. This is confirmed by increasing a Hall field or R gradually: Drastic changes arise around $R \sim 0.1$; there the Hall plateaus disappear and a sizable amount of Hall current is seen to flow in the bulk. This leads to observation (v).

2

Figure 2 shows another demonstration of current redistribution via disorder. There it is clearly seen that electron states residing on the edges of the disordered region ("bulk edges") support a considerable amount of Hall current per state and that a small number of states in the inner bulk carry an even larger amount.

Observation (v) suggests a possible mechanism for the breakdown of the QHE. We study the competition of the Hall field with disorder in detail in the next section.

4. Field-induced breakdown of the QHE

To simulate electrons in the bulk of a realistic sample we have considered electron states residing over a disordered domain of size $\approx 70\ell \times 30\ell$, where 360 random impurities with $|s_i| \leq s_{\max}$ are distributed [14]. The main observation drawn from our numerical experiment is that, in the regime of dense impurities $N_{\mathrm{imp}}/N_{\mathrm{state}} > 1$ (which presumably applies to realistic samples), the number of localized states, N^{loc}, decreases exponentially with $|E_y|$ and obeys an approximate scaling law written in terms of

$$R \equiv e\ell\,|E_y|/(s_{\max}\,\omega) \propto |E_y|/(s_{\max}\,B^{3/2}). \tag{3}$$

See the $N^{\mathrm{loc}} - R$ plots for the uppermost subband in Fig. 3(b); they stay approximately the same for a wide variation in magnetic field $B = B_0/\kappa$ with $\kappa = 1 \sim 4$. See also the $J_x^{\mathrm{Hall}} - N_{\mathrm{v}}$ plots in Fig. 3(a) for various choices of R. Actually the data refer to the $n = 0$ subband, but the way the plateaus shrink with increasing R turns out virtually the same for higher $n = 1, 2, 3$ subbands as well. This shows that the field-to-disorder ratio R is a good measure to express field-induced delocalization of electron states.

The underlying physical picture revealed by this scaling behavior of N^{loc} is that the electron states remaining localized in the near-breakdown regime are always governed by the same set of impurities of large strength $|s_i| \sim |s|_{\max}$; that is, one always encounters the same set of localized states prior to breakdown. We have confirmed this by observing that suppressing weaker impurities (e.g., $|s_i| < 0.5\,s_{\max}$) entails no essential change in the behavior of N^{loc} for large R.

This picture offers a simple explanation for the observed $E_y^{\mathrm{cr}} \propto B^{3/2}$ law of the critical breakdown field. (The fact that this scaling law is observed for samples differing in carrier density and mobility [11-13] would be attributed to relatively small variations in disorder-strength $|s|_{\max} \sim O(0.1)$ for a variety of samples.) It is also consistent with the observation of Cage *et al.* [9] that the breakdown of the dissipationless current is spatially inhomogeneous. Finally, it is a natural consequence of the present *intra*subband process that the magnitude of E_y^{cr} falls within the observed range of a few hundred V/cm, one order of magnitude smaller than what intersubband processes, such as Zener tunneling [10], typically predict.

5. Concluding remark

As for current redistribution via disorder, discussed in Sec. 3, it is essential to separate the (slow) Hall-current component j_{Hall} from the (fast) equilibrium current j_x. A way to achieve such separation in experiment is to add a small alternating component to an injected direct (or slowly-alternating) current. Information on the Hall-current distribution would then be obtained by detecting the alternating component in the Hall-potential distribution responding to it.

Acknowledgments

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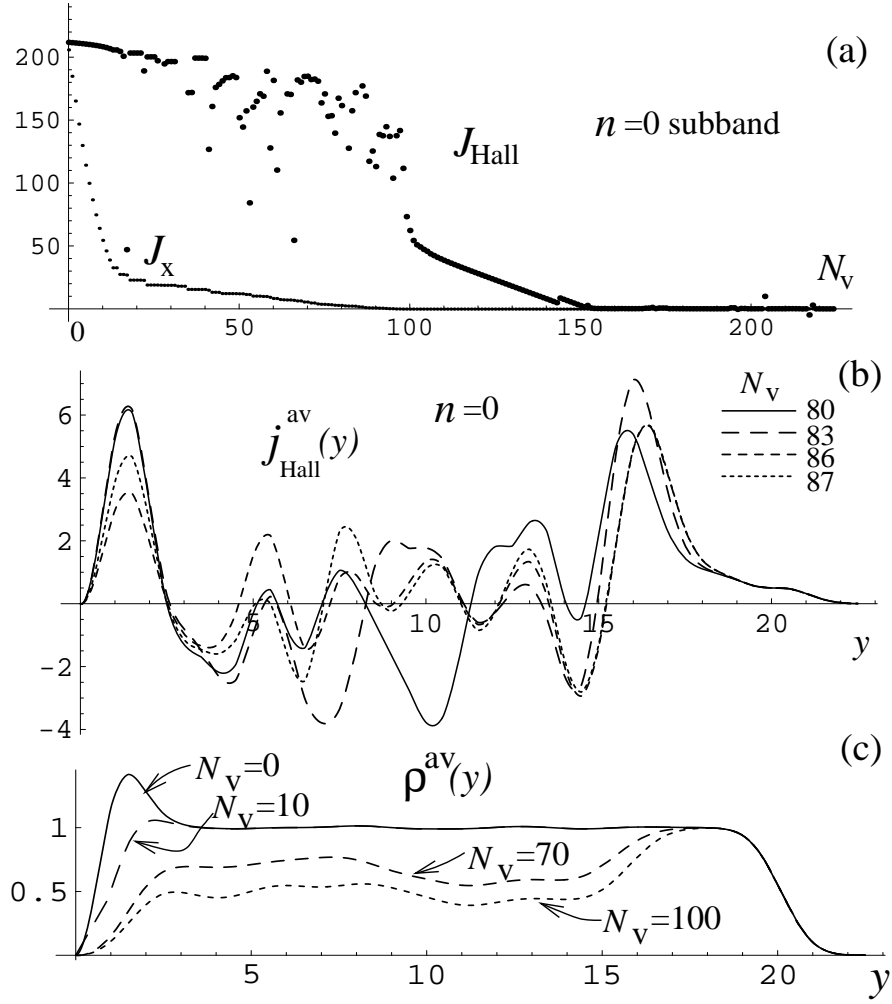FIG. 1.   (a) Total currents J_{Hall} and J_x (on different scales) vs vacancies N_{v}. (b) Hall-current distribution $j_{\text{Hall}}^{\text{av}}(y)$ [in units of $-(e^2/2\pi\hbar)\,\delta E_y$] in the plateau regime. (c) Density distribution.
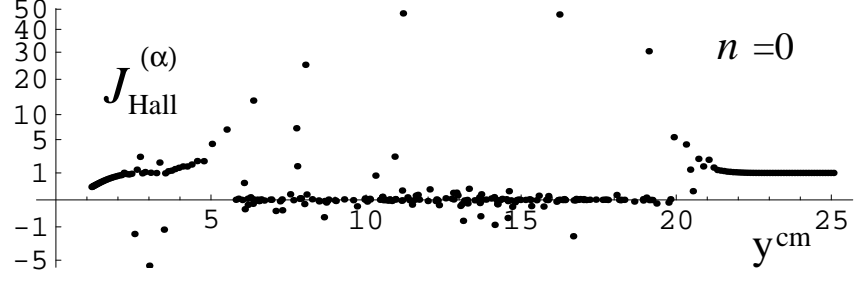
FIG. 2. Net amount of Hall current per state [on a square-root scale] as a function of cen-
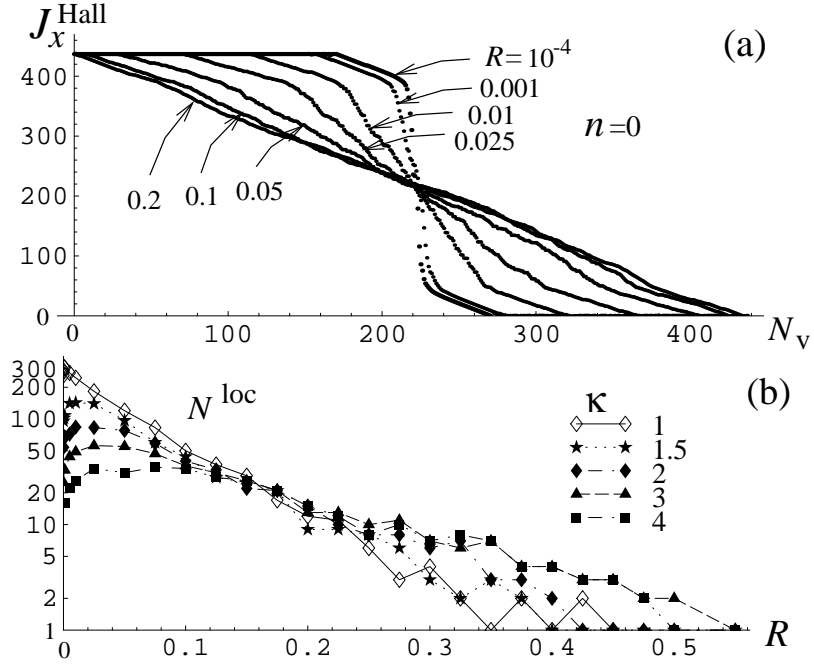ter-of-mass position y^{cm} for each state. Impurities lie over $5\ell \le y \le 20\ell$.

FIG. 3.   (a) J_x^{Hall} vs N_{v} for $R = 0.0001 \sim 0.2$.(b) $\log N^{\mathrm{loc}}$ vs R for $B = B_0/\kappa$ with $\kappa = 1 \sim 4$.